Design Guide for Insulated Wall Panels using Beam-Spring Modeling Techniques, Part II **2025-09-17**

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Part 2 – Detailed Application of the Beam-Spring Method:

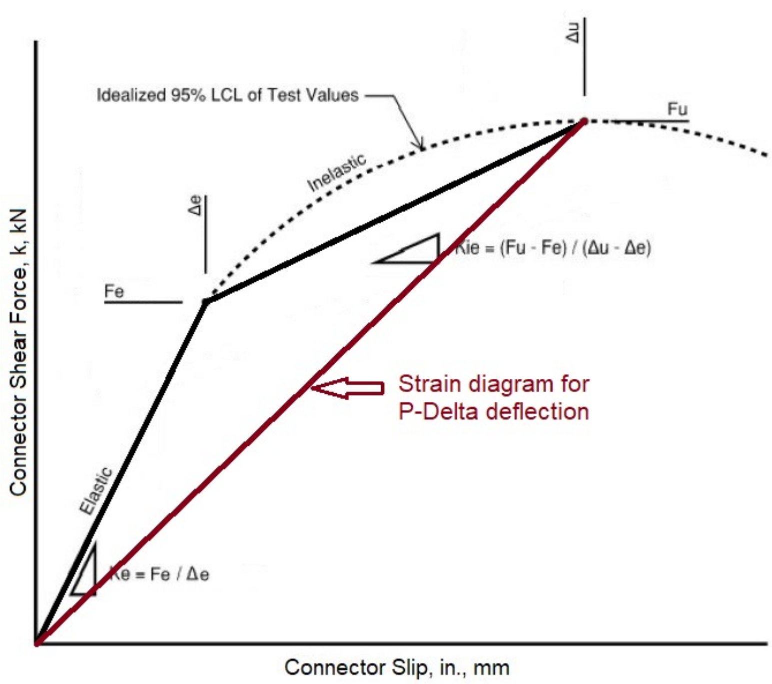
2.1 Modeling considerations

2.1.1 Secondary effects (PΔ):

Load-bearing panels should be checked for secondary moments due to PΔ effects. If the frame analysis program has a 2nd order analysis option, this should be selected. If not, then the analysis needs to be run iteratively, adding the node deflections manually and re-running until an approximate convergence is reached. The only value we are interested in is the lateral deflection due to PΔ effects.

The first step is to apply all loads with eccentricities and perform a beam-spring primary analysis to find the initial deflection at each node. These values are then applied as input to the PΔ second order analysis. The PΔ analysis only uses self-weight and axial loads applied at the member centroid, no vertical eccentricities. The final run will use the initial deflection (Δi) combined with the PΔ deflection (Δp). Δtotal = Δi + Δp

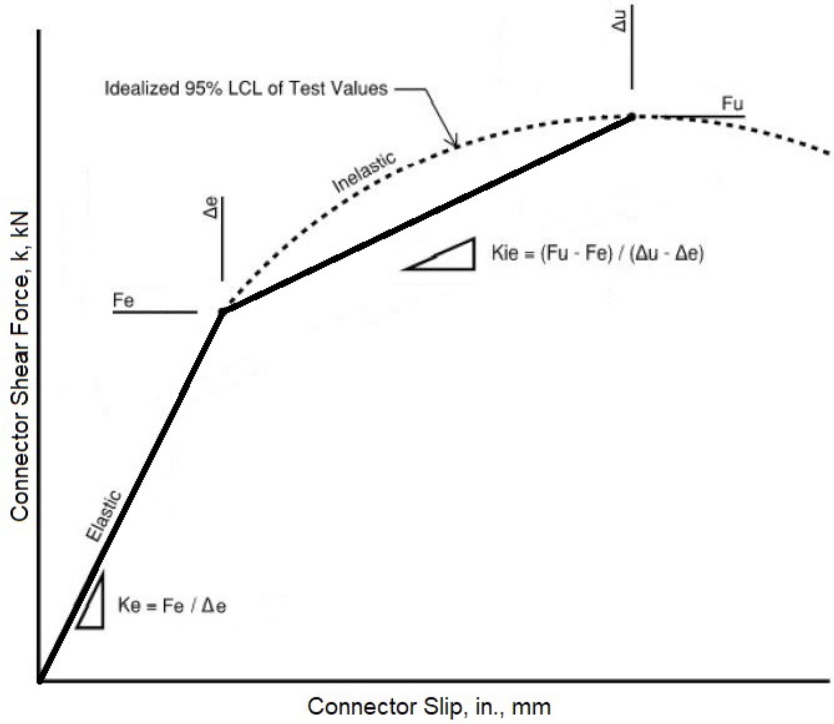
Slender, **mild-reinforced** load-bearing panels should follow the **ACI Slender Wall method** (ref). PΔ deflections are maximized by using a fully-cracked moment of inertia and cracked wythe cross-section area. In addition, the wythe connectors are assumed to have yielded, with a stiffness input as shown:



Wythe connector strain used for PΔ analysis of slender, mild-reinforced panels

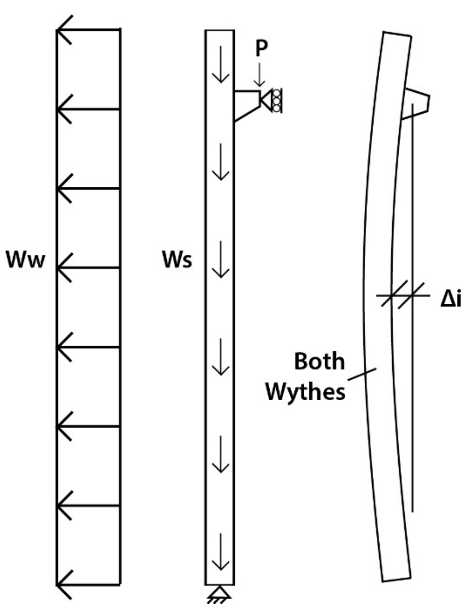
PΔ connector stiffness is therefore assumed to equal Fu/Δu for the PΔ analysis.

Prestressed panels only: PΔ effects are modeled differently for prestressed panels. Wythe connector stiffness follows the bi-linear curve:



Wythe Connector Slip (strain) vs. force graph (LCL = Lower Control Line, Ke = elastic stiffness, Kie = inelastic stiffness)

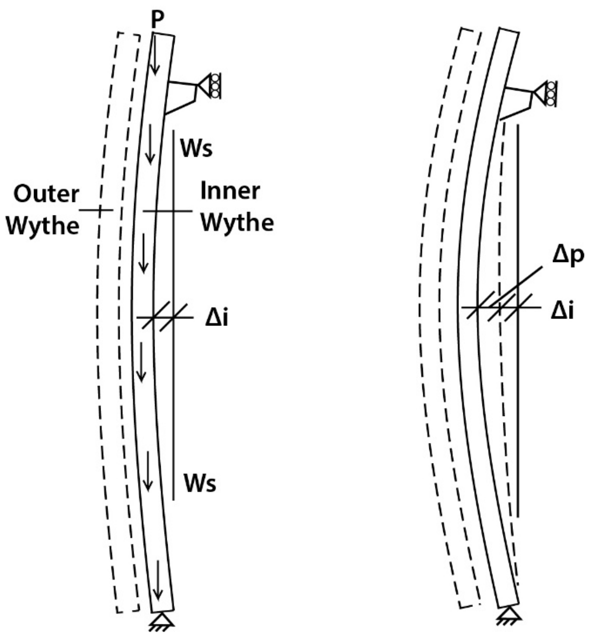
For simplicity, the connector stiffness could be limited to the elastic range (Ke) if slip forces are low enough to allow it. Also, if extreme fiber concrete wythe stress stays below the cracking stress, then a gross moment of inertia can be used for the concrete wythes. For higher stresses, ACI 318-19 Table 24.2.3.5b should be used to calculate I-effective values for the wythes.



***Primary run(s)** to find initial deflection (Δi) due to applied loads. Ww = Wind load, Ws = panel self-weight, P = applied Dead + Roof load.*

Prestressed: Use I-gross for the first Primary run to find initial extreme fiber stresses and connector slip. For subsequent runs, use I-effective if the wythe is cracked. Connector stiffness, Ke = Fe/Δe (elastic range). Since moments will redistribute if a member wythe cracks, the loading could be gradually increased and I-effective readjusted after each run until the full load is applied.

Mild reinforced: Use I-gross for the first Primary run to find initial extreme fiber stresses and connector slip. Use I-cracked and A-cracked for subsequent runs. Connector stiffness, Ku = Fu/Δu (inelastic). This will maximize deflections per the ACI Slender Wall method.

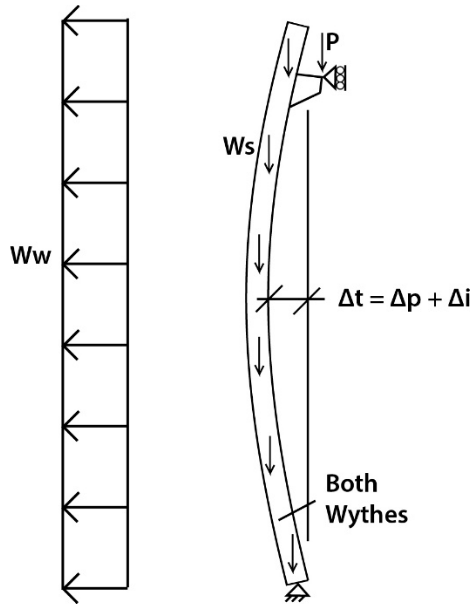


Secondary effects run(s) *to find deflection (Δp) due to PΔ effects alone.*

For the Secondary Effects (PΔ) runs, wind and any other lateral loads are ignored (these loads were accounted for in the Primary runs). Both Ws, panel self-weight, and P = Dead + Roof load are applied at the centroid of the inner wythe only, regardless of actually eccentricity. Note that Ws is the weight of both wythes applied to the centroid of the inner wythe only. The base rocker is replaced with a pinned connection at the base of the inner wythe. The outer wythe is connected and contributes to stiffness but has no load on it – it “floats”. The only moments that should be generated will be those due to PΔ effects, otherwise the model will not converge. The node coordinates are set to the Initial deflection, Δi, for the first run, increasing as Δp is added for each consecutive run, until Δp converges.

Prestressed: Use I-gross or I-effective if wythe is cracked. Connector stiffness, Ke = Fe/Δe (elastic range). Inelastic connector stiffness could be used, up to 0.75*Fu (see Section 2.2.1 below), if the analysis software supports bilinear (inelastic) behavior.

Mild reinforced: Use I-cracked and A-cracked. Connector stiffness, Ku = Fu/Δu (inelastic). This will maximize deflections per the ACI Slender Wall method.



Combined run *using Δt = Δi + Δp. Full loads and eccentricities are applied to both wythes, as with the Primary run, only with an initial bow equal to* Δt.

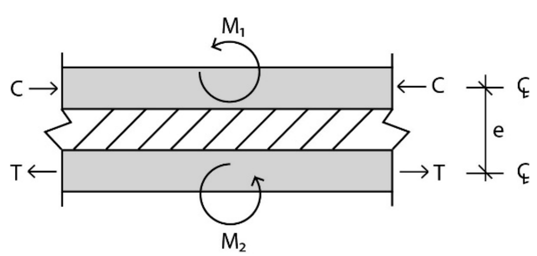
Prestressed: Use I-gross or I-effective if the wythe is cracked. Connector stiffness, Ke = Fe/Δe (elastic range). Inelastic connector stiffness could be used, up to 0.75*Fu, if the analysis software supports bilinear behavior.

Mild reinforced: Use I-cracked and A-cracked. Connector stiffness, Ku = Fu/Δu (inelastic).

As noted in Part I, the analysis will predict the axial tension (T) or compression (C) in each concrete wythe, as well as the moment (M_1, M_2) taken by each wythe and the connector shear (V) at each row along the member length. The member capacity will be a combination of the tension-compression moment couple between the wythes and moments taken by each wythe individually. The higher the moment couple is in relation to the wythe moments, the greater the degree of composite action.

Actual connector slip (Δ) can be found by rearranging the slip equation from Part I: $\Delta = (Ve^3)/(12EI)$, where e is the center to center distance between the wythes, E is the connector modulus of elasticity and I is the connector moment of inertia (the same values that were input to the beam-spring frame analysis).

A **Wythe run** is then used to find final wythe moments and flexural stresses. I-gross and A-gross are used with the final deflection in order to attract moment to each wythe. Compare this moment with the moment capacity of the wythe.



Total moment = $T * e + M_1 + M_2$*, assuming* $T = C$

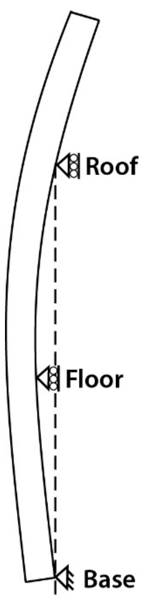
2.1.2 Thermal loads:

Differential temperature strains can be input to the model as a Strain load. For example, a 30-degree Fahrenheit (F) difference between the wythes would produce a strain equal to C * F, where C is the coefficient of thermal expansion of concrete per degree F. In the example case, C * F = 0.000006 * 30 = 0.00018 inches per inch. Add this Strain load to the wythe that is warmer, or, conversely, add a -0.00018 negative strain to the cooler wythe. Whichever approach is used doesn't matter to the beam-spring model.

2.1.3 Prestress Camber:

Prestress camber, or bow, induced by differential prestress must be handled differently from Thermal. This is because the bow is present before the roof and intermediate support connections are made, unlike with Thermal. A beam-spring prestress strain run is required to find this bow before running the Primary runs. Determine the prestress strains using the final prestress force and the concrete modulus of elasticity of each wythe. One can either input the strains from both wythes to the model or just the differential strain to one of the wythes. This assumes that the strands are concentric to each wythe. If not, then the moments introduced by the eccentric strands also need to be input as a moment couple at the wythe ends.

Record the node or joint displacements from this run to find the prestress bow. These coordinates are then input as the initial conditions for the Primary beam-spring run (see 2.1.1 above).



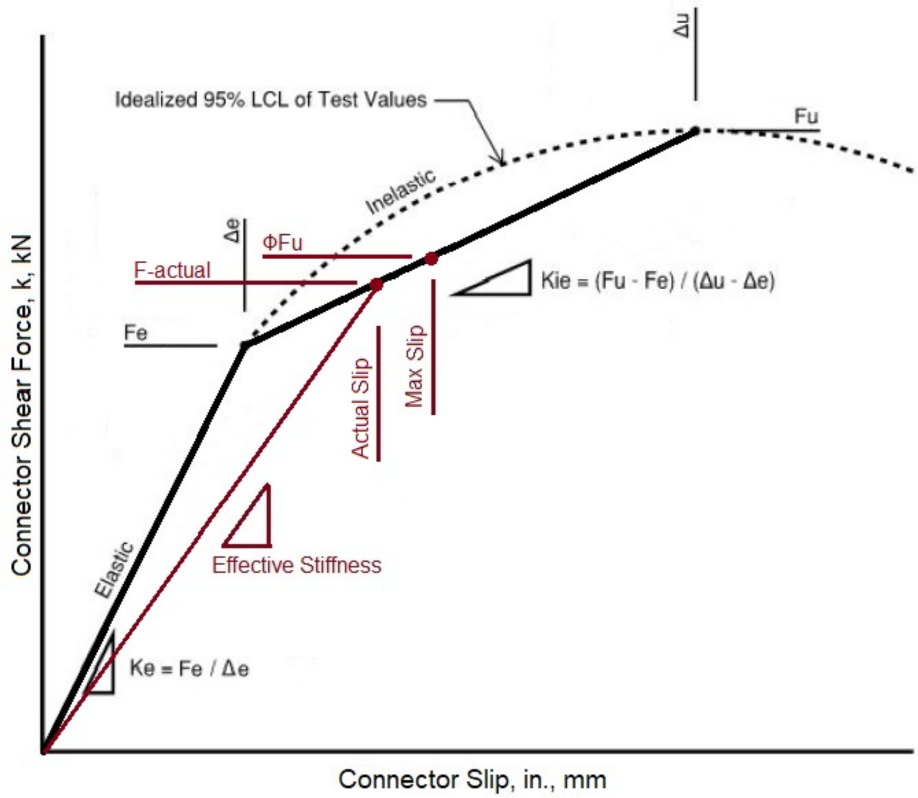
Prestress-induced camber (exaggerated) with an intermediate floor connection and a parapet.

2.2 Interpreting the analysis results:

The values obtained from the beam-spring analyses are used to evaluate the ultimate capacity and service level performance of the insulated wall panel.

2.2.1 The three ultimate strength checks:

First, the wythe connector nodes should be checked to ensure that the connector forces do not exceed the allowed maximum ultimate horizontal shear value. According to the PCI 150 Specification, Section 4.1.3, the connector shear force should not exceed 0.75 * Fu. (The connector manufacturer may have a lower limit for this Φ value.) Connector node forces can be obtained from the first Primary run, which used gross properties. If the connector shear force is greater than Fe, then the connector stiffness should be reduced to an effective stiffness as shown below and the analysis run again:



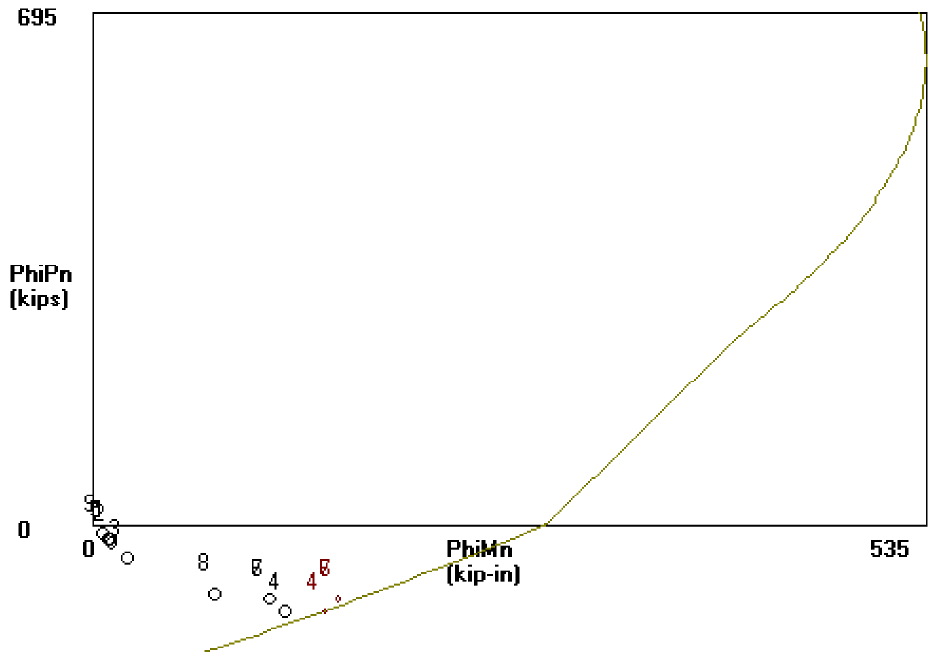
The connector shear force should not exceed ΦFu.

Check that the connector shear forces in the **Combined** run do not exceed 0.75*Fu.

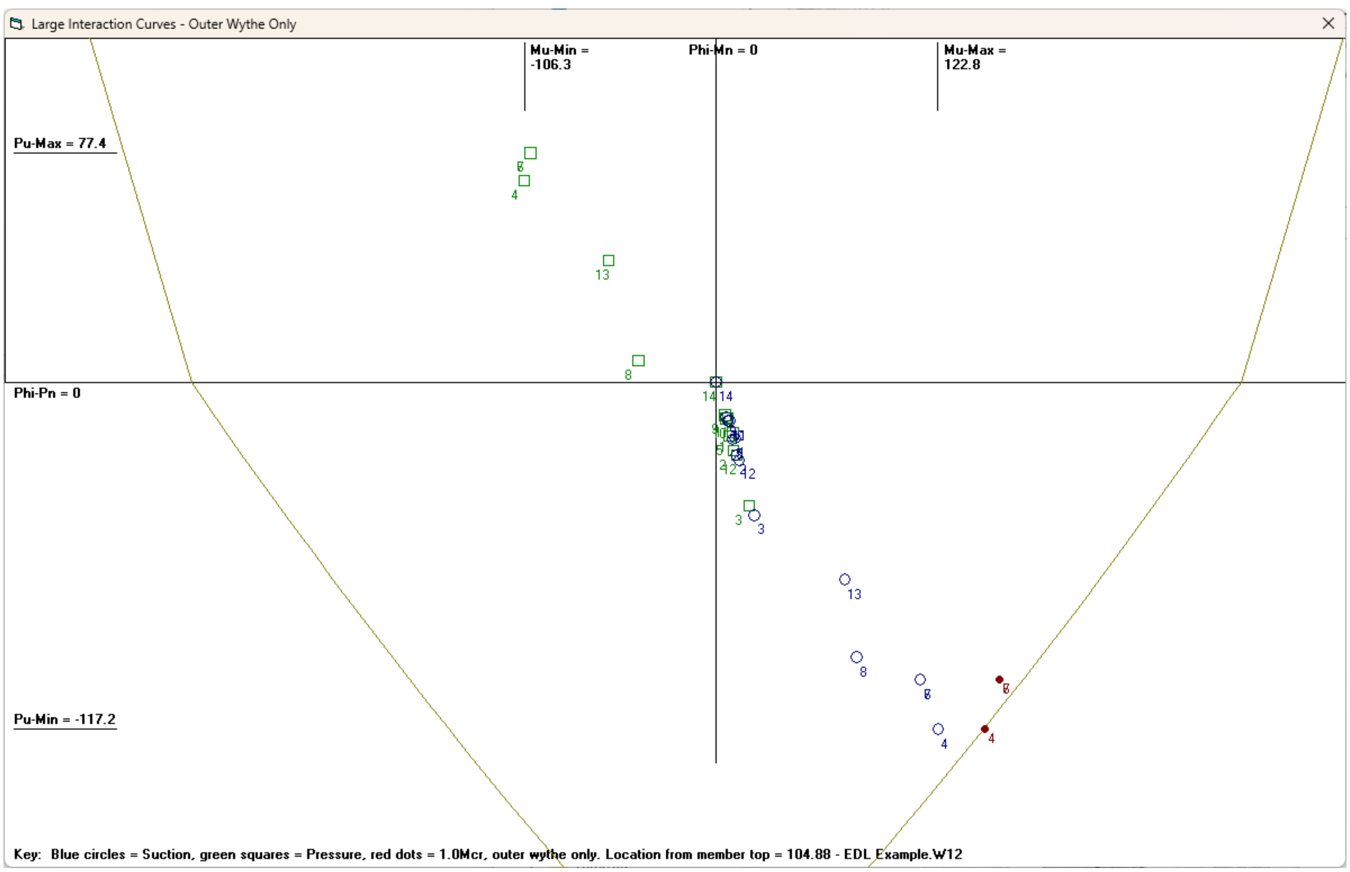
Second, the moment capacity of each wythe should be checked. To do this, another beam-spring run is needed. This **Wythe** run applies the full Δt bow from the **Combined** run but uses I-gross and A-gross for the concrete wythes. This will attract moment to the concrete wythes, since they are still assumed to be uncracked. An interaction curve for each wythe needs to be calculated and plotted. Follow the procedure in the PCI Handbook, Section 5.9.1 "Strength Design of Precast Concrete Compression Components", which is based on strain compatibility.

The cracking moment, 1.0*Mcr, should also be plotted for non-prestressed members. Section 4.1.4 of the PCI 150 Design Specification states that:

"Flexure-governed precast, nonprestressed concrete insulated wall panels shall satisfy φMn ≥ 1.0 Mcr at the section along the wall panel length where the onset of flexural cracking is expected, unless φMn ≥ 2.0 Mu."



Conventional interaction curve for outer wythe suction with all load cases plotted. Note that the critical points are located where the wythe is in axial tension (Pu < 0). Cracking moments, 1.0Mcr, are shown as red dots.



Combined outer wythe interaction curve for both wind suction and pressure load cases. Here, wind pressure exerts a compression force in the outer wythe, while suction creates axial tension in the wythe.

If the moment capacity of a wythe is exceeded, this could be considered as only a partial failure (for equal wythes only), as there is still the tension-compression couple between the wythes to consider (the third check). Unequal wythes are covered below in Section 2.2.1.

Third, if the axial tension in each wythe is less than the reinforcing 0.9*Fy or 0.9*Fpu (ϕTn) in that wythe, then the panel passes the ultimate capacity test for flexure. Use the ***Combined*** run for this check, as it minimizes the moment taken by each wythe and maximizes the moment couple between the wythes.

2.2.2 Service Stresses and Deflections:

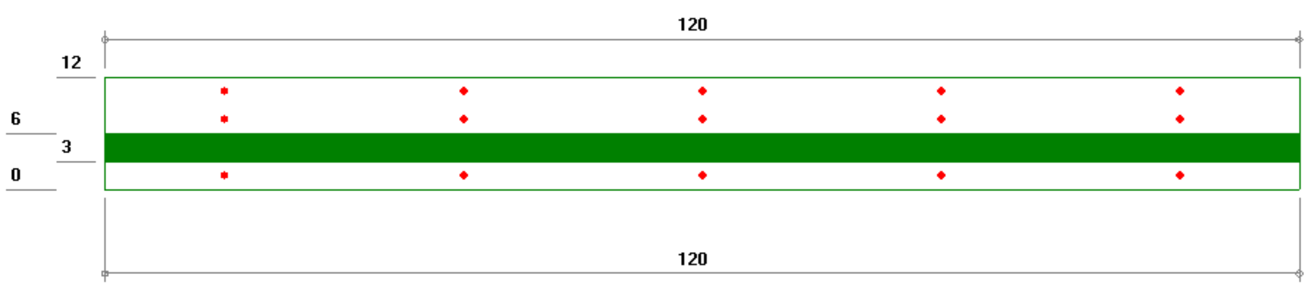
Service **stresses** in the concrete wythe extreme fibers can be found by repeating the above Primary and Secondary runs with gross section properties, using Service loads instead of Ultimate loads. Tension stress = f = M/S + P/A - fps, where M is the moment in the concrete wythe at the node in question, S is the section modulus, P is the axial tension in the wythe node, A is the gross wythe concrete area and fps is the prestress stress in the wythe, if any. Note that P would be negative for axial compression.

The same beam-spring Service stress runs can be used to find service **deflections** by listing node or joint displacements.

2.2.3 Unequal wythes:

As noted in Part I, the beam-spring method is uniquely suited to analyzing panels with unequal wythe thicknesses. The thicker wythe will take most of the moment force, with the other wythe in a supporting role. As such, the second strength check in Section 2.2.1 above is critical. The maximum moment in the thick wythe is compared to the wythe moment capacity at that point. In this case, if moment capacity is exceeded, it is not a partial failure, but a full failure, and more reinforcing should be added.

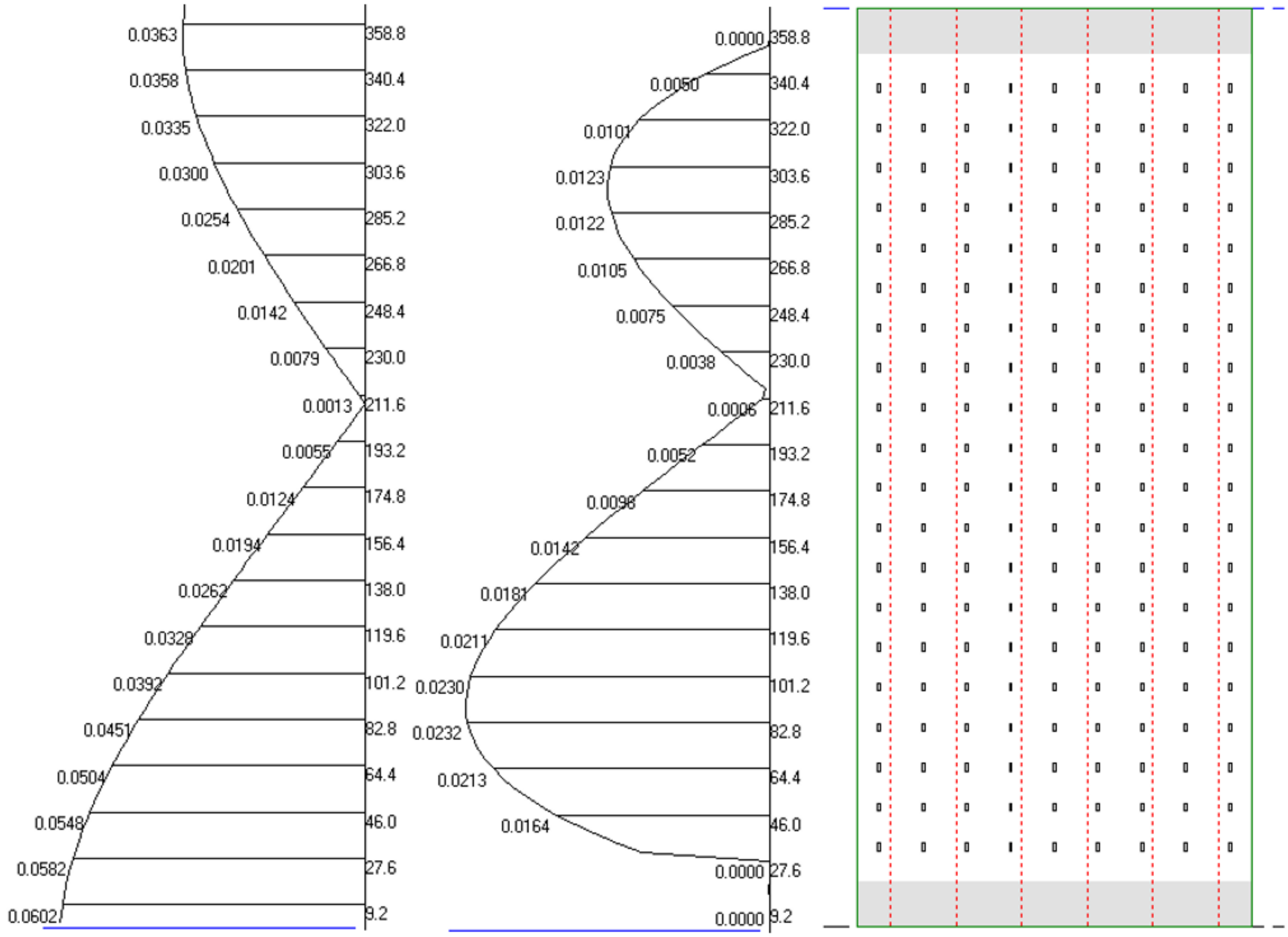
If the moment capacity of the thin wythe is exceeded, then an axial tension check is needed (the third check in Section 2.2.1).



A section view of an unequal wythe panel with 3" insulation.

2.2.4 Solid zones:

To model solid zones with beam-spring, the member properties (Area (A), Modulus of Inertia (I), and Modulus of Elasticity (E)) for wythe connector members in the solid zone areas are replaced with much, much larger values (see 1.7.2 in Part I). For our example with 16" connector spacing, the Area would be 16*16 = 256 in^2. The Modulus of Inertia would equal $b*d^3/12$ or $16^4/12$ = 5461 in^4, and the Modulus of Elasticity would equal the E of the concrete, 4415 ksi in this case, assuming 6000 psi concrete. The actual values are not important, as long as they are large enough to result in a slip that is essentially zero.



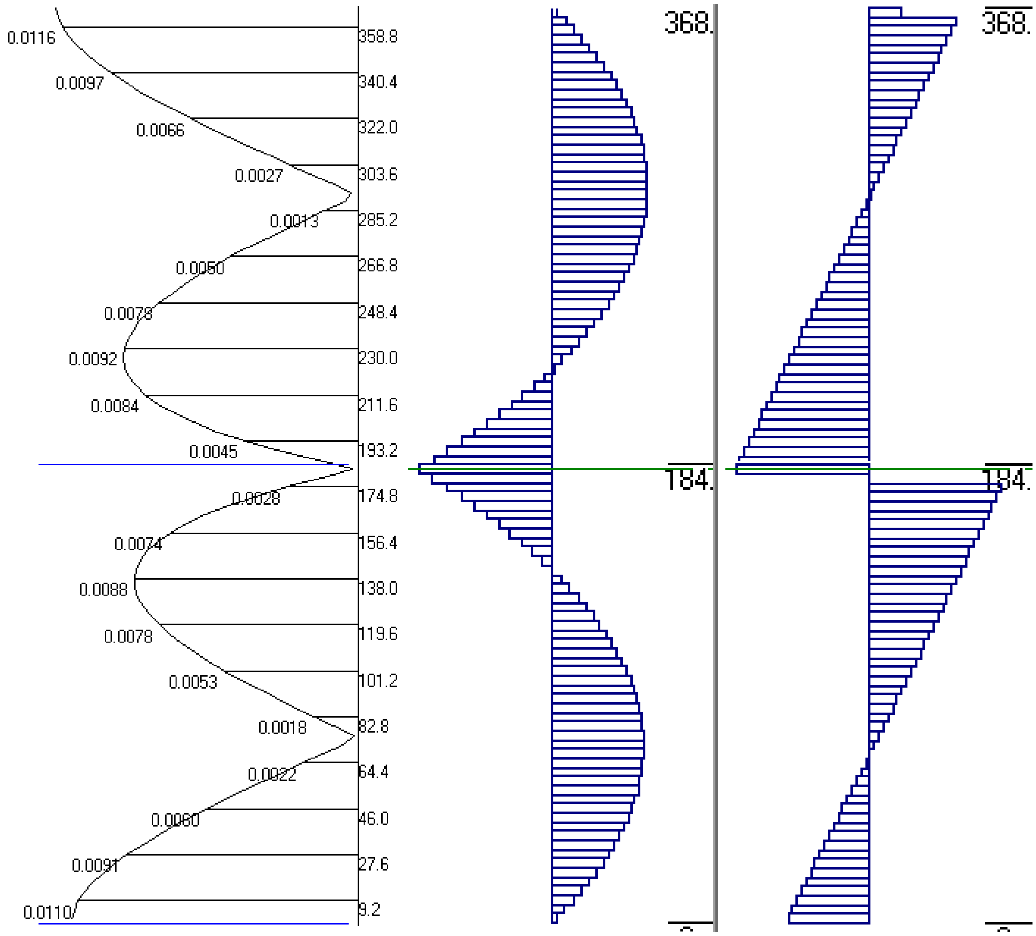
Left: Connector slip without solid zones.   Right: Connector slip with top and bottom solid zones.

As can be seen from these sample connector slip diagrams, slip resolves to zero at the solid ends. The maximum slip in the connectors is also much lower, which helps satisfy the first ultimate strength check (see 2.2.1). The second and third ultimate wythe checks still need to be examined (wythe flexure will decrease but wythe axial tension will increase).

Horizontal shear at solid zones needs to stay below 80 psi for the solid zones to be counted, per ACI 318-19 Table 16.4.4.2. When the ACI shear phi factor is added, this reduces to 60 psi. (Phi is 0.75 for shear). If this shear is exceeded, there are two options: 1. Ignore the contribution of the solid zone for ultimate loads, or 2. Add tied reinforcing in the solid zone to bridge the wythes sufficient to take the full horizontal shear. (See Section 1.5.1 in Part I for more information.)

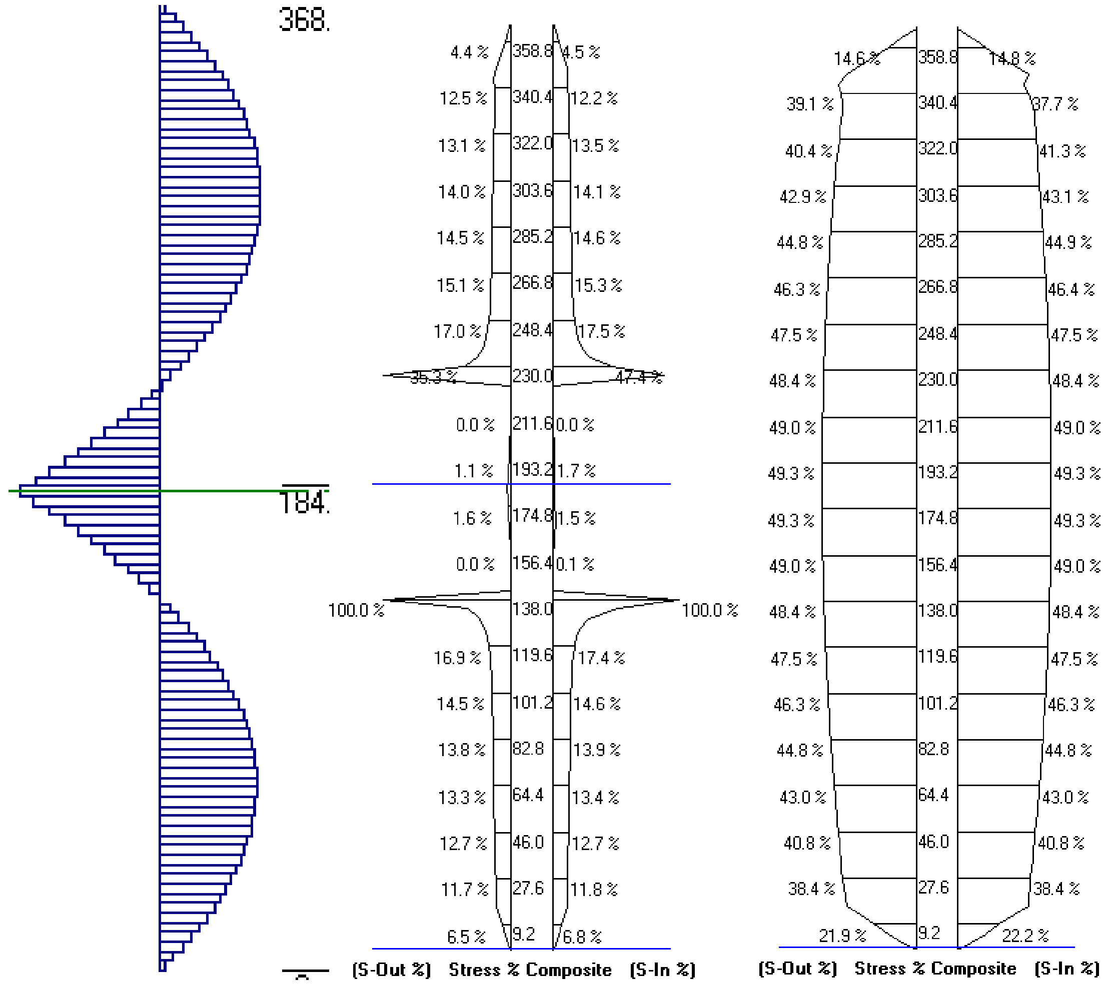
2.2.5 Multi-span behavior:

Adding a mid-height support changes the wythe connector slip behavior dramatically. In this example, subject to lateral wind load, the intermediate support is at 184” and the top support is at 368”:



Left: Connector slip with mid-ht support   Middle: Moment diagram   Right: Shear diagram

When the slip goes to zero, the composite action also disappears at that point. Note that the zero slip points correspond to the points of shear reversal, as well as the points of maximum moment. Theoretically, according to the beam-spring analysis, the applied moment at these points is resisted almost solely by the individual wythe flexural capacities, as there is no longer a significant composite moment couple. As discussed in Part I, the percent composite method does not account for this behavior. The percent composite method is therefore not applicable when there are intermediate supports.



Left: Moment diagram *Middle: Stress % composite* *Right: Simple-span stress % composite*

In this example, the percent composite for stresses goes to zero near the midspan support, compared to 49% for a comparable simple span. Insulation stiffness may help increase composite action, but this effect is not accounted for with the beam-spring model.

Part III, A Sample Problem:

In Part III, an analysis of this sample problem will be run from start to finish, using a basic plane frame analysis program that is similar to STAAD, as well as MASTAN2, an open-source program.